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The Structure and Uniqueness of Generalized Solutions of the Spherically Symmetric Einstein-Scalar Equations*

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Abstract. In a previous paper we proved the global existence of generalized solutions of the spherically symmetric Einstein-scalar field equations in the large. In this paper we study the regularity properties of the spacetime and the scalar field corresponding to a generalized solution. We also prove a uniqueness theorem which shows that a generalized solution is an extension of a classical solution.

Section 0. Introduction

In [1] we began the study of the global initial value problem for the Einstein-scalar equations $R_{\mu\nu} = 8\pi\partial_\mu\phi\partial_\nu\phi$ in the spherically symmetric case. In terms of a retarded time coordinate u and a radial coordinate r, the spacetime metric has the form

$$ds^2 = -e^{2\nu}du^2 - 2e^{\nu+\lambda}dudr + r^2 d\Sigma^2,$$

where $d\Sigma^2$ is the metric of the standard 2-sphere. The problem is best formulated in terms of the function $h := \partial(r\phi)/\partial r$. If f is a function of u and r we denote by $\bar{f}$ the mean value function of f:

$$\bar{f}(u,r) := \frac{1}{r}\int_0^r f(u,r')\,dr'.$$

Defining then

$$g := \exp\left[-4\pi\int_r^\infty (h-\bar{h})^2\frac{dr}{r}\right], \quad D := \frac{\partial}{\partial u} - \tfrac{1}{2}\bar{g}\frac{\partial}{\partial r},$$

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