Commun. Math. Phys. 109, 563–589 (1987)

Communications in Mathematical Physics

Large-time Behavior of Solutions of the Discrete Boltzmann Equation

Shuichi Kawashima*

Department of Mathematics, Nara Women's University, Nara 630, Japan

Abstract. Large-time behavior of solutions of the one-dimensional discrete Boltzmann equation is studied. Under suitable assumptions it is proved that as time tends to infinity, the solution approaches a function which is constructed explicitly in terms of the self-similar solutions of the Burgers equation and the linear heat equation.

1. Introduction

The one-dimensional discrete Boltzmann equations is written in the form (see Appendix)

$$\frac{\partial F_i}{\partial t} + v_i \frac{\partial F_i}{\partial x} = \frac{1}{\alpha_i} \sum_{j,k,l=1}^{m} (A_{kl}^{ij} F_k F_l - A_{ij}^{kl} F_i F_j), \quad i = 1, \ldots, m. \tag{1.1}$$

Here $F_i = F_i(t,x) \geqq 0$ denotes the mass density of gas particles with the velocity v_i (real constant) at time $t \geqq 0$ and position $x \in \mathbb{R}$. The coefficients α_i are positive constants. Also, A_{kl}^{ij} are nonnegative constants satisfying

$$A_{lk}^{ij} = A_{kl}^{ij} = A_{kl}^{ji}, \quad A_{kl}^{ij} = A_{ij}^{kl} \tag{1.2}$$

for any $i, j, k, l = 1, \ldots, m$. In order to exclude the trivial case, we may assume that

$$A_{kl}^{ij} \neq 0 \quad \text{for some} \quad i, j, k, l = 1, \ldots, m. \tag{1.3}$$

We rewrite (1.1) in the vector form. Put $F = {}^t(F_1, \ldots, F_m) V = \mathrm{diag}(v_1, \ldots, v_m)$ and $Q(F,G) = {}^t(Q_1(F,G), \ldots, Q_m(F,G))$, where each $Q_i(F,G)$ is defined by

$$Q_i(F,G) = \frac{1}{2\alpha_i} \sum_{j,k,l} \{A_{kl}^{ij}(F_k G_l + F_l G_k) - A_{ij}^{kl}(F_i G_j + F_j G_i)\}, \tag{1.4}$$

Present address: Department of Applied Science, Faculty of Engineering, Kyushu University 36, Fukuoka 812, Japan