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Nonlinear Parabolic Stochastic Differential Equations with Additive Colored Noise on $R^d \times R_+$: A Regulated Stochastic Quantization

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Abstract. We prove the existence of solutions to the nonlinear parabolic stochastic differential equation

$$(\partial/\partial t - \Delta)\varphi = -V'(\varphi) + \eta_c$$

for polynomials V of even degree with positive leading coefficient and η_c a gaussian colored noise process on $R^d \times R_+$. When η_c is colored enough that the gaussian solution to the linear problem has Hölder continuous covariance, the nongaussian processes are almost surely realized by continuous functions. Uniqueness, regularity properties, asymptotic perturbation expansions and nonperturbative fluctuation bounds are obtained for the infinite volume processes. These equations are a cutoff version of the Parisi-Wu stochastic quantization procedure for $P(\varphi)_d$ models, and the results of this paper rigorously establish the nonperturbative nature of regularization via modification of the noise process. In the limit $\eta_c \to$ gaussian white noise we find that the asymptotic expansion and the rigorous bounds agree for processes corresponding to the (regulated) stochastic quantization of super-renormalizable and small coupling, strictly renormalizable scalar field theories and disagree for nonrenormalizable models.

1. Introduction and Overview

This work is motivated by the stochastic quantization procedure proposed by Parisi and Wu [1]. In this approach, the euclidean field measure for fields on R^d with action functional S

$$d\mu(\varphi) = \exp(-S[\varphi]) \prod d\varphi(x) / \int \exp(-S[\varphi]) \prod d\varphi(x) \tag{1.1}$$

is considered as the formal stationary probability distribution of the random process defined by the stochastic differential equation (Langevin equation)

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