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Semi-Infinite Ising Model

I. Thermodynamic Functions and Phase Diagram in Absence of Magnetic Field

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Abstract. For the semi-infinite Ising model in two or more dimensions, we prove analyticity properties of the surface free energy and map out the phase diagram in the absence of an external magnetic field. We prove that this phase diagram contains critical lines where the parallel and/or the transverse correlation lengths diverge. The critical exponent, $\nu_\perp$, of the transverse correlation length is shown to be equal to the exponent ν of the Ising model on an infinite lattice. In a second paper, these results will be used to analyze the wetting transition.

1. Introduction

We consider a binary system in the two-phase region, with phases + and −. We suppose that the system is in the − phase. If we insert a wall, which adsorbs preferentially the + phase, there is formation of a film of the + phase between the wall and the bulk phase. There is a *partial wetting* of the wall when the thickness of the film is microscopic, and *complete wetting* when the thickness is macroscopic. The *wetting transition* is the transition from partial wetting to complete wetting. This phenomenon can be analyzed in the Ising model. Let us consider the Ising model on $\mathbb{Z}^d$, with Hamiltonian

$$-\sum_{\langle ij\rangle} K\sigma(i)\sigma(j)-\sum_i \lambda\sigma(i), \tag{1.1}$$

where $\langle ij\rangle$ indicates a pair of points $\{i,j\}$ such that $|i-j|=1$. We insert a wall by setting $\sigma(i)=1$, for all $i=(i^1,\ldots,i^d)\in\mathbb{Z}^d$ with $i^d\leqq 0$. In this way we get a semi-infinite model on the sublattice

$$\mathbb{L}=\{i\in\mathbb{Z}^d;\ i^d>0\}=\mathbb{Z}^{d-1}\times\mathbb{Z}^+ \tag{1.2}$$

with coupling constant K, external field λ and boundary field K. We generalize the model by admitting an arbitrary boundary field h and by choosing a coupling constant J for the interaction of two spins inside the first layer of $\mathbb{L}$,

$$\Sigma=\{i\in\mathbb{L};\ i^d=1\}\cong\mathbb{Z}^{d-1}. \tag{1.3}$$