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Construction and Borel Summability of Infrared Φ_4^4 by a Phase Space Expansion

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Abstract. We construct the thermodynamic limit of the critical (massless) φ^4 model in 4 dimensions with an ultraviolet cutoff by means of a "partly renormalized" phase space expansion. This expansion requires in a natural way the introduction of effective or "running" constants, and the infrared asymptotic freedom of the model, i.e. the decay of the running coupling constant, plays a crucial rôle. We prove also that the correlation functions of the model are the Borel sums of their perturbation expansion.

Introduction

This paper extends the methods of constructive field theory to treat strictly renormalizable asymptotically free situations. We study the infrared behavior of massless Φ_4^4 with an ultraviolet cutoff as one of the simplest of these situations. We use an approach which has its source in the work of Glimm and Jaffe [1] on the ultraviolet limit of Φ_3^4. The basic tool of this paper is a kind of "phase space expansion" [1–5]; many of its features were already presented in [6], where it was used to control the "infrared superrenormalizable" $(\nabla\Phi)_3^4$ model. It consists of scaled cluster expansions with effective parameters. It had however to be further improved to apply to strictly renormalizable theories and this resulted also in a number of simplifications. We give to the present expansion the name PRPSE (for partially renormalized phase space expansion). The methods developed in [7] to control the large orders of perturbation theory for Φ_4^4, and in [8] to exploit rigorously asymptotic freedom at the level of Feynman graphs played an important role in the genesis of this paper; in particular they helped convince us that constructive field theory could attack non-superrenormalizable situations. In fact the results of [7] can be recovered using the present version of the phase space expansion [9, 10].

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