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Ondelettes and Phase Cell Cluster Expansions, A Vindication*

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Abstract. Y. Meyer has recently developed a particularly useful o.n. basis for $L^2(R^d)$. Expansions using this basis, i.e. expansions into "ondelettes" or "wavelets," have yielded important new results in soft and hard analysis. The expansion into ondelettes of a boson scalar field naturally leads to phase cell cluster expansions, a formalism already developed by the authors using other related bases. Adoption of ondelettes expansions into the phase cell program gives improvements of some extant results, and excises an early error.

Ondelettes lend more elegance to the phase cell cluster expansion of ϕ_3^4, and to us are a vindication of the fundamental nature of this approach. This provides more promise for future developments.

1. Ondelettes

We consider compatible lattices $\mathscr{L}^r$ in R^d of edge size $L_r = (1/2^r)$, $r = 0, \pm 1, \pm 2, \ldots$. We now state some of the properties of the o.n. basis $\{\psi_k\}$ developed by Y. Meyer associated to these lattices [5].

1) Each ψ_k is associated to some cube, in some $\mathscr{L}^r$, $r = r(k)$. There are the same number of ψ_k associated to each cube. Given the set of ψ_k associated to any cube, the set of ψ_k associated to any other cube are the natural dilation, translation, and multiple of these ψ_k. More exactly, there are a number of functions $f_1, \ldots, f_\alpha$ such that the ψ_k associated to cube γ in $\mathscr{L}^r$, $r = r(\gamma)$, are

$$L_{r(\gamma)}^{-d/2} f_i\left(\frac{x - x^{(\gamma)}}{L_{r(\gamma)}}\right), \quad i = 1, \ldots, \alpha,$$

where $x^{(\gamma)}$ is the center of γ.

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