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Time-Delay and Lavine's Formula

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Abstract. Lavine's results on time-delay ([10]) is extended to higher dimensional Schrödinger operators.

1. Introduction

In [10], Lavine proved the existence of a quantity called time-delay and gave its representation formula which we call "Lavine's formula," for one-dimensional Schrödinger operators. The aim of this paper is to extend them to n-dimensional Schrödinger operators.

We consider Schrödinger operators:

$$H = H_0 + V(x); \quad H_0 = -\Delta$$

on $\mathscr{H} = L^2(\mathbb{R}^n)$, and we suppose that the potential V satisfies

Assumption (V). $V(x) = V_1(x) + V_2(x)$, and there exists a constant $\varepsilon > 0$ such that
(i) $V_1(x)$ is a C^∞-function and for any α,

$$\left| \left(\frac{\partial}{\partial x} \right)^\alpha V_1(x) \right| \leqq C_\alpha (1 + |x|)^{-1-\varepsilon-|\alpha|}; \tag{1.1}$$

(ii) the multiplication operator by $V_2(x)$ is compact from $H^2(\mathbb{R}^n)$ to $L^{2,2+\varepsilon}(\mathbb{R}^n)$.

$L^{2,\alpha}(\mathbb{R}^n) = \{\phi \in L^2_{\mathrm{loc}}(\mathbb{R}^n) : (1 + |x|)^\alpha \phi \in L^2(\mathbb{R}^n)\}$ is the weighted L^2-space of order α. Then, as is well-known, H is self-adjoint; the wave operator defined by

$$W_\pm = \operatorname*{s-lim}_{t \to \pm\infty} \exp(itH) \exp(-itH_0)$$

exists and is complete: Ran $W_\pm = \mathscr{H}^{ac}(H)$; hence the scattering operator defined by $S = W_+^* W_-$ is unitary.

For $R > 0$, let X_R be a multiplication operator defined by

$$X_R = X_R(x); \quad X_R(x) = X(|x|/R); \quad 0 \leqq X(x) \leqq 1;$$
$$X \in C_0^\infty(\mathbb{R}); \quad X(x) = 1 \quad \text{if} \quad |x| \leqq 1, = 0 \quad \text{if} \quad |x| \geqq 2. \tag{1.2}$$