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Infrared Asymptotic Freedom for the Pseudoscalar Yukawa Model at the Critical Point

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Abstract. We consider the pseudoscalar Yukawa model on a four dimensional Euclidean lattice. The fermions are integrated out giving an effective boson interaction. We show that for weak coupling this interaction is in the class covered by the Gawedzki–Kupiainen renormalization group analysis. It follows that one can adjust parameters so that the flow of the renormalization group is toward a free massless boson theory. This establishes the existence of the critical theory and controls its long distance behavior.

I. Introduction

There has recently been substantial progress made in developing rigorous non-perturbative renormalization schemes for quantum field theory ([1,2,4,7–9,12–14]). Much of this work uses Wilson's formulation of the renormalization group.

In this approach one begins with a lattice field theory on a fine lattice and attempts to take the continuum limit by studying effective interactions on coarser lattices obtained by block spin transformations. This is the basic short distance or ultraviolet problem. The problem can also be formulated as the problem of studying the scaling limit for a critical field theory on a unit lattice. In this version a first step is obtaining the critical theory, a question which is also of interest for statistical mechanics. Thus one arrives at a class of long-distance or infrared problems which still carry many of the basic problems of renormalization. The technique for controlling the infinite volume limit is again to use block spin transformations, this time to get effective interactions in smaller volumes.

In this paper we study the infrared problem for a critical pseudoscalar Yukawa theory. Our approach uses the renormalization group analysis of Gawedzki and Kupiainen ([10–13]). The remainder of this introduction is devoted to a precise description of their results, and in the balance of the paper we show how they can be applied to our model.

Let L be an odd integer and let Λ be a four-dimensional unit toroidal lattice