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The Decomposition Property and Equilibrium States of Ferromagnetic Lattice Systems

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Abstract. We discuss classical lattice gas models with a finite number of different particles and ferromagnetic type interaction between them. We make the set of particle types into a finite abelian group and explore the algebraic structure of such a system. We present a criterion for using the Peierls argument to establish the existence of phase transitions. In the case of a cyclic group of order equal to the product of different prime numbers we obtain a complete description of all periodic Gibbs states at low temperatures.

1. Introduction

One of the main tasks of statistical mechanics is to describe the family of equilibrium states of the system for a given interaction, external parameters, and temperature. This problem is addressed here in the context of certain classical lattice gas models. Namely, at each site of the lattice there is a variable which can take on a finite number of values. One may think of these as different species which can occupy the lattice sites or $2l+1$ orientations of a spin l particle. The particles or spins interact through many body potentials. Central in the analysis of low temperature behavior of such systems is the notion of a ground state and the so-called Peierls condition. A ground state is a configuration of particles with a minimal potential energy per lattice site. A model satisfies the Peierls condition if the creation of an "island" of one periodic ground state in a "sea" of another periodic ground state leads to an increase of energy which is greater than some fixed positive constant times the length of the boundary of the "island". In the case of models which have a finite number of periodic ground states and satisfy the Peierls condition there is a complete theory due to Pirogov and Sinai [1, 2] (see also the review article by Slawny [3]). Namely, the phase diagram at low temperatures is obtained by perturbation of the zero

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