

# Renormalization Group Approach to Lattice Gauge Field Theories

## I. Generation of Effective Actions in a Small Field Approximation and a Coupling Constant Renormalization in Four Dimensions

Tadeusz Bałaban

Department of Physics, Lyman Laboratory, Harvard University,  
Cambridge, MA 02138, USA

**Abstract.** We study four-dimensional pure gauge field theories by the renormalization group approach. The analysis is restricted to small field approximation. In this region we construct a sequence of localized effective actions by cluster expansions in one step renormalization transformations. We construct also  $\beta$ -functions and we define a coupling constant renormalization by a recursive system of renormalization group equations.

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### 0. Introduction. Formulations of Results

In this paper we continue our study of the ultraviolet stability problem for lattice approximations of gauge field models. We consider here the renormalization group approach to four-dimensional pure gauge field theories. We restrict the study to a part of the problem, namely we want to understand how to generate the effective actions in a small field approximation, and how to perform a coupling constant renormalization by a system of recursive renormalization group equations (Callan-Symanzik equations).

The renormalization group approach we use here follows the ideas of Wilson [74, 75]. This approach was developed by the author for superrenormalizable gauge field theories in the papers [4–16], see especially [4–6] for an explanation of ideas and an introduction to techniques. The renormalization group approach