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An L^p-Theory for the n-Dimensional, Stationary, Compressible Navier-Stokes Equations, and the Incompressible Limit for Compressible Fluids. The Equilibrium Solutions

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Abstract. In this paper we study the system (1.1), (1.3), which describes the stationary motion of a given amount of a compressible heat conducting, viscous fluid in a bounded domain Ω of R^n, $n \geqq 2$. Here $u(x)$ is the velocity field, $\rho(x)$ is the density of the fluid, $\zeta(x)$ is the absolute temperature, $f(x)$ and $h(x)$ are the assigned external force field and heat sources per unit mass, and $p(\rho, \zeta)$ is the pressure. In the physically significant case one has $g = 0$. We prove that for small data (f, g, h) there exists a unique solution (u, ρ, ζ) of problem (1.1), $(1.3)_1$, in a neighborhood of $(0, m, \zeta_0)$; for arbitrarily large data the stationary solution does not exist in general (see Sect. 5). Moreover, we prove that (for barotropic flows) the stationary solution of the Navier-Stokes equations (1.8) is the incompressible limit of the stationary solutions of the compressible Navier-Stokes equations (1.7), as the Mach number becomes small. Finally, in Sect. 5 we will study the equilibrium solutions for system (4.1). For a more detailed explanation see the introduction.

1. Introduction

In this paper we study the system

$$\begin{cases} -\mu \Delta u - \nu \nabla \operatorname{div} u + \nabla p(\rho, \zeta) = \rho[f - (u \cdot \nabla) u], \\ \operatorname{div}(\rho u) = g, \\ -\chi \Delta \zeta + c_v \rho u \cdot \nabla \zeta + \zeta p'_\zeta(\rho, \zeta) \operatorname{div} u = \rho h + \psi(u, u), \quad \text{in} \quad \Omega, \\ u_{|\Gamma} = 0, \zeta_{|\Gamma} = \zeta_0, \end{cases} \tag{1.1}$$

in a bounded open domain Ω in R^n, for arbitrarily large $n \geqq 2$. It is assumed that Ω lies (locally) on one side of its boundary Γ, a C^2 manifold. Here,

$$\psi(u, u) = \chi_0 \sum_{i,j=1}^{n} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)^2 + \chi_1 (\operatorname{div} u)^2, \tag{1.2}$$