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# A Triangulation of Moduli Space from Light-Cone String Theory

Steven B. Giddings[1],* and Scott A. Wolpert[2],**

[1] Joseph Henry Laboratories, Princeton University, Princeton, NJ 08544, USA
[2] Department of Mathematics, University of Maryland, College Park, MD 20742, USA

**Abstract.** We show that scattering diagrams for closed strings in light-cone string theory provide a single cover of the moduli space of Riemann surfaces.

Since the early days of string theory physicists have known that modular invariance, or more accurately, invariance under the mapping class group, plays a fundamental role in the theory. Modular invariance is especially relevant to such consistency issues as unitarity and finiteness. In the Polyakov approach [1] to string theory modular invariance is seen directly: the integrand in correlation functions or the partition function is modular invariant in the critical dimension, and the integration region is taken to be one copy of the fundamental domain of the mapping class group, or equivalently moduli space. This approach to string theory is very elegant, but such basic properties as unitary are not manifest.

There has traditionally been another way of approaching string theory, and that is the interacting string picture as formulated in the light-cone gauge [2]. In this approach Lorentz invariance is not manifest, but unitarity is apparent since we work only with physical states. However, it has not been proven that the light-cone picture is equivalent to the Polyakov picture. One of the steps in this proof is settling the question whether the light-cone formalism reproduces the integration over a single copy of the moduli space for a Riemann surface of arbitrary genus. Some physicists have implicitly assumed that the answer to this question is yes. In fact, it is hard to imagine that the Polyakov theory is correct if it isn't equivalent to the light-cone formulation[1] since the light-cone formulation manifests essential properties like unitarity. If the light-cone formulation didn't reproduce the integration over moduli space we would have to add or subtract by hand the integration over the rest of moduli space; such tinkering could easily ruin the nice properties of the theory.

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