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Scaling Relations for 2*D*-Percolation

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Abstract. We prove that the relations

$$\beta=\frac{2\nu}{\delta+1},\quad \gamma=2\nu\frac{\delta-1}{\delta+1},\quad \Delta=2\nu\frac{\delta}{\delta+1},\quad \text{and}\quad \eta=\frac{4}{\delta+1},$$

hold for the usual critical exponents for 2*D*-percolation, provided the exponents δ and ν exist. Even without the last assumption various relations (inequalities) are obtained for the singular behavior near the critical point of the correlation length, the percolation probability, and the average cluster size. We show that in our models the above critical exponents have the same value for approach of p to the critical probability from above and from below.

1. Introduction

It is widely believed (see for instance [6, 10, 26]; also [25] for critical exponents in general) that various quantities in percolation behave like powers of $|p-p_c|$ as p approaches the critical probability p_c. To express these conjectures we use the following notation for site percolation on a periodic graph $\mathscr{G}$ in $\mathbb{R}^d$ (see [13, Chaps. 1–3] for a precise description of the terminology). P_p denotes the probability measure according to which all sites of $\mathscr{G}$ are, independently of each other, occupied (vacant) with probability p (respectively $q:=1-p$). E_p denotes expectation with respect to the probability measure P_p. W is the occupied cluster of a certain preselected site w_0, which will be taken to be the origin $\mathbf{O}$ whenever possible.

$$\# W = \text{number of sites in } W,$$

$A \rightsquigarrow B$ means that there exists an occupied path on $\mathscr{G}$ from some site in A to some site in B,

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