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Field Theoretical Construction of an Infinite Set of Quantum Commuting Operators Related with Soliton Equations

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Abstract. The quantum version of an infinite set of polynomial conserved quantities of a class of soliton equations is discussed from the point of view of naive continuum field theory. By using techniques of two dimensional field theories, we show that an infinite set of quantum commuting operators can be constructed explicitly from the knowledge of its classical counterparts. The quantum operators are so constructed as to coincide with the classical ones in the $\hbar \to 0$ limit ($\hbar$; Planck's constant divided by 2π). It is expected that the explicit forms of these operators would shed some light on the structure of the infinite dimensional Lie algebras which underlie a certain class of quantum integrable systems.

1. Introduction

The existence of an infinite set of polynomial conserved quantities in involution to each other under a certain Poisson bracket is one of the most characteristic features of classical soliton theories [1] in $1+1$ dimensions, which are the best understood nonlinear field theories. The infinite set of conserved quantities is the cornerstone for the complete integrability of classical systems. The situation is the same for quantum theories. Therefore a natural question arises whether the classical infinite dimensional symmetry survives quantization. Namely, do we get an infinite set of quantum commuting operators which reduces to the classical one in the limit of $\hbar \to 0$ ($\hbar$; the Planck constant divided by 2π), or do we not, due to some anomalies caused by the high nonlinearity of the interaction? Usually this problem is investigated system by system in terms of the quantum inverse method [2] or by the transfer matrix approach based on a lattice [3]. In such approaches, however, the field theoretical aspects are made rather obscure behind a strong algebraic structure like the Yang-Baxter algebra.

In this connection we should mention one of the successful examples of the quantum inverse method, the nonlinear Schrödinger equation,

$$i\psi_t + \psi_{xx} - \kappa\psi^\dagger\psi\psi = 0 .$$