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A Strictly-Positive Mass Theorem*

P. F. Yip

Department of Mathematics, University of California, San Diego, La Jolla, CA 92093, USA

Abstract. We show that the ADM 4-momentum of an isolated gravitational system (spatially asymptotically flat spacetime) satisfying the dominant energy condition cannot be null-like unless it is flat. Together with the positive mass theorem, this implies that the ADM 4-momentum of an isolated gravitational system must be strictly time-like.

Introduction

The total mass of an isolated gravitational system is, by now, well known to be non-negative, thanks finally to theorems of Schoen and Yau [1] and Witten [2]. More precisely, it was shown that for a spatially asymptotically flat spacetime satisfying the dominant energy condition, its ADM 4-momentum must be non-space-like. This result was anticipated by and, in turn, confirmed the physical intuition that an isolated gravitational system is stable. However, it is interesting to observe that our physical intuition of what an isolated gravitational system should be, actually demands a stronger result, namely, the total mass must be strictly positive (equivalently, the ADM 4-momentum must be strictly time-like).

Let us first consider the situation of the Maxwell fields. One can certainly find solutions (plane waves) whose 4 momenta are null-like[1]. But these waves are infinite in extent and in no way can be called isolated. To form an isolated wave (i.e. a wave packet), one has to superpose plane waves with different momenta; the total 4-momentum of such a wave packet must, of course, be strictly time-like. Since general relatively in many ways resembles Maxwell theory, general features of basically the same physical concept should be common to both theories. Thus, we expect that the total 4-momentum of an isolated gravitational system must be strictly time-like. There is a caveat, though, general relativity is nonlinear. Could there exist a situation in general relativity where the nonlinearities can "glue"

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[1] Here, of course, the total 4-momentum is infinite. What we mean is that the field is $\alpha e^{i(Et-\mathbf{p}\cdot\mathbf{x})}$ with $E^2-\mathbf{p}\cdot\mathbf{p}=0$