Commun. Math. Phys. 108, 647–651 (1987)

Communications in
Mathematical
Physics

An Example of Absence of Turbulence for Any Reynolds Number: II*

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Abstract. We study a viscous incompressible fluid moving in a two dimensional flat torus $[0,L]\times[0,2\pi]$, $L<2\pi$. We show a set of external forces for which the stationary state is attractive for any Reynolds number R. Moreover, the size of this set and the basin of attraction are independent of R.

In a previous paper [1] we have considered a viscous incompressible fluid moving in a two dimensional flat torus $[0,L]\times[0,2\pi]$, $L\leqq 2\pi$. We have shown an external force $\mathbf{f}_0$ for which there is a globally attractive stationary state for any Reynolds number R. Moreover, we proved that this stability property holds also for a neighbourhood of $\mathbf{f}_0$ of *size depending on* R (and vanishing for $R\to\infty$). In the present paper we demonstrate that actually for $L<2\pi$ the size of this neighbourhood is *independent of* R.

The Navier-Stokes equations governing the motion are

$$\partial_t \mathbf{u}+(\mathbf{u}\cdot\nabla)\mathbf{u}=-\nabla p+\mathbf{f}+\nu\Delta\mathbf{u}\,,\qquad \mathbf{u}(0)=0\,, \tag{1}$$

$$\partial_x u_x+\partial_y u_y=0\,, \tag{2}$$

$$\int_D \mathbf{u}d\mathbf{x}=0\,;\qquad \int_D \mathbf{f}dx=0\,,$$

$$D=[0,L]\times[0,2\pi]\,;\qquad \mathbf{x}=(x,y)=x\mathbf{c}_1+y\mathbf{c}_2\in D\,,$$

where $\mathbf{u}(x,t)$ is the velocity, $p(x,t)\in R^+$ the pressure, $\nu>0$ the viscosity, $\mathbf{f}(x)$ the external force. All functions involved are periodic of period L in x and 2π in y.

We introduce the vorticity $\omega=\partial_x u_y-\partial_y u_x$.

Equation (1) becomes

$$\partial_t\omega+(\mathbf{u}\cdot\nabla)\omega=F+\nu\Delta\omega\,, \tag{3}$$

where

$$F=\partial_x f_y-\partial_y f_x\,.$$

* Research partially supported by Italian CNR and Ministero della Pubblica Istruzione