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Supersymmetric Path Integrals

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Abstract. The supersymmetric path integral is constructed for quantum mechanical models on flat space as a supersymmetric extension of the Wiener integral. It is then pushed forward to a compact Riemannian manifold by means of a Malliavin-type construction. The relation to index theory is discussed.

Introduction

An interesting new branch of mathematical physics is supersymmetry. With the advent of the theory of superstrings [1], it has become important to analyze the quantum field theory of supersymmetric maps from R^2 to a manifold. This should probably be done in a supersymmetric way, that is, based on the theory of supermanifolds, and in a space-time covariant way as opposed to the Hamiltonian approach. Accordingly, one wishes to make sense of supersymmetric path integrals. As a first step we study a simpler case, that of supersymmetric maps from R^1 to a manifold, which gives supersymmetric quantum mechanics. As Witten has shown, supersymmetric quantum mechanics is related to the index theory of differential operators [2]. In this particular case of a supersymmetric field theory, the Witten index, which gives a criterion for dynamical supersymmetry breaking, is the ordinary index of a differential operator. If one adds the adjoint to the operator and takes the square, one obtains the Hamiltonian of the quantum mechanical theory. These indices can be formally computed by supersymmetric path integrals. For example, the Euler characteristic of a manifold M is supposed to be given by integrating e^{-L}, with

$$L=\frac{1}{2}g_{ij}(\phi)\dot{\phi}^i\dot{\phi}^j-\frac{1}{2}g_{ij}(\phi)\psi^{i\dagger}\frac{D}{dt}\psi^j-\frac{1}{8}R_{ijkl}(\phi)\psi^{i\dagger}\psi^j\psi^{k\dagger}\psi^l,$$

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