Commun. Math. Phys. 108, 483–488 (1987)

Communications in
Mathematical
Physics

Instability of the Boundary in the Billiard Ball Problem

Andrea Hubacher
ETH Zürich, CH-8092 Zürich, Switzerland

Abstract. We consider the billiard ball problem in the interior of a plane closed convex C^1 curve which is piecewise C^2. If the curvature has a discontinuity, then the boundary is unstable, i.e. no caustics exist near the boundary. However, in the interior there can exist caustics, as we show by an example.

We consider the billiard ball problem in the interior of a plane closed convex C^1 curve. We assume that the second derivatives of the curve exist and are continuous except in finitely many points, where the limits of the second derivatives exist from both sides, but where the curvature is discontinuous. Furthermore we assume the curvature to be strictly positive and uniformly bounded. This situation is illustrated by a convex boundary consisting of circular arcs matching with their tangents (see [4]).

Denote the boundary curve of the billiard table by C and its total length by L. The billiard ball problem can be described in the two coordinates arc length s and angle t between outgoing billiard ray and positive tangent direction in a point of reflection at the boundary. Associating to such a pair of coordinates (s, t) the corresponding pair (s_1, t_1) of the next reflection point (see Fig. 1) gives rise to a homeomorphism from the annulus $(\mathbb{R}/L\mathbb{Z}) \times [0, \pi]$ to itself, which we will call the *billiard map* (see [1]).

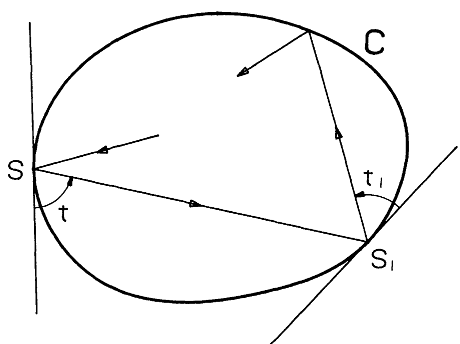

Fig. 1. The billiard map