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Effective Action for the Yukawa$_2$ Quantum Field Theory

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Abstract. Using a rigorous version of the renormalization group we construct the effective action for the Y_2 model. The construction starts with integrating out the bosonic field which eliminates the large fields problem. Studying the so-obtained purely fermionic theory proceeds by a series of convergent perturbation expansions. We show that the continuum limit of the effective action exists and its perturbation expansion is Borel summable.

I. Introduction

The Yukawa$_2$ quantum field theory has a long history. Its existence was first proved by Glimm and Jaffe [1, 2] and Schrader [3] within the Hamiltonian, or Minkowski space framework. By constructing the Euclidean Fock space and proving the Feynman-Kac formula, Osterwalder and Schrader [4] established the equivalence between the Hamiltonian and the Euclidean formalisms. The crucial step towards Euclidean construction of the model was done by Seiler [5]. He integrated out the Fermi field and proved that the resulting determinant was integrable with respect to the free bosonic measure. This paper was followed by [6, 7] where the stability bounds were proved and by [8, 9] where the thermodynamic limit was constructed and the Wightman axioms were verified. Renouard [10] showed subsequently that the theory was Borel summable, and Balaban and Gawedzki [11] proved the existence of two phases in the chiral Yukawa$_2$ theory.

In the present work we propose a new approach to the Yukawa$_2$ model which consists, in a sense, in reversing Seiler's approach. We start the analysis with integrating out the bosonic field, and study the resulting purely fermionic theory with a non-local quartic interaction. The inspiration for doing this comes from the remarkable papers [12, 13] where the effective action for the Gross-Neveu model has been constructed. The analysis of [12, 13] is in the spirit of the renormalization group (RG) program (for review, see [14–16]) combined with the old observation by Caianello [17] that regularized fermionic perturbation theory converges. This convergence is because the Feynman graphs of a given order appear with either sign, owing to Fermi-Dirac statistics. The resulting cancellations between the