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Semiclassical Resonances Generated by a Closed Trajectory of Hyperbolic Type

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Abstract. We determine all the resonances in certain rectangular regions of the complex plane of the Schrödinger operator $-h^2\Delta + V$ when $h\to 0$, under the assumption that the set of trapped points of energy 0 for the classical flow form a closed trajectory and that the corresponding Poincaré map is hyperbolic.

0. Introduction

In this paper we consider a semiclassical differential operator P on $\mathbb{R}^n$ with analytic coefficients, which satisfies all the general assumptions of [6, Sect. 8]. Let $p(x,\xi)$ be the principal symbol in the sense of h-pseudodifferential operators. [The most important special case is, of course, when $P=-h^2\Delta+V(x)$. Then $p=\xi^2+V(x)$.] We assume that

$$p(x,\xi)=0 \;\Rightarrow\; dp\neq 0\,. \tag{0.1}$$

In the appendix of this paper, we give some generalities concerning the flow of $H_p=\sum p'_{\xi_j}\partial_{x_j}-p'_{x_j}\partial_{\xi_j}$ either in $p^{-1}([-\varepsilon_0,\varepsilon_0])$ or in $p^{-1}(0)$. For $\varrho\in T^*\mathbf{R}^n$, let $]T_-(\varrho),T_+(\varrho)[\ni t\mapsto \exp tH_p(\varrho)$ be the maximal classical trajectory. Here T_+ and $-T_-$ are lower semicontinuous functions of ϱ with values in $]0,+\infty]$. We define the outgoing tail and the incoming tail by

$$\tilde{\Gamma}^0_\pm=\{\varrho\in p^{-1}(0);\ \exp tH_p(\varrho)\not\to\infty,\ \text{as}\ t\to T_\mp(\varrho)\}\,. \tag{0.2}$$

In the appendix we show among other things, that $K^0=\tilde{\Gamma}^0_+\cap\tilde{\Gamma}^0_-$ is a compact set. Our next assumption is then:

$$\begin{gathered}K^0 \text{ is (the image of) a simple closed trajectory}\\ \gamma^0:[0,T^0]\to p^{-1}(0)\quad [\text{satisfying } \gamma^0(0)=\gamma^0(T^0)]\,.\end{gathered} \tag{0.3}$$

Let p^0 be the corresponding linearized Poincaré map. It is a symplectic automorphism of the normal space of γ^0 in $p^{-1}(0)$ at the point $\gamma^0(0)$, defined as the differential of the smooth map $H^0\to H^0$, obtained by following the flow of H_p once around γ^0. Here $H^0\subset p^{-1}(0)$ is some smooth hypersurface intersecting γ^0