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Supermanifold Cohomology and the Wess-Zumino Term of the Covariant Superstring Action

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Abstract. The cohomology theory of supermanifolds is developed. Its basic properties are established and simple examples given. The Wess-Zumino term in the Green-Schwarz covariant superstring action is interpreted as a nontrivial class in the "supersymmetric cohomology" of flat superspace. A quotient supermanifold with nontrivial topology reflecting this class is constructed. It is shown that there is no topological quantization condition for the coefficient of the Wess-Zumino term. The superstring differs from conventional sigma models in this respect because its action is Grassmann-valued and its group manifold (superspace) is noncompact.

1. Introduction

The covariant action discovered by Green and Schwarz for the superstring [1] is not simply the obvious generalization of the supersymmetric particle action. In addition to a kinetic term which does generalize the particle action, there is an additional term which is necessary in order to obtain a somewhat mysterious local supersymmetry. This supersymmetry is needed in order to gauge away unphysical degrees of freedom and establish the equivalence with the light-cone gauge action. Henneaux and Mezincescu, and independently Martinec, subsequently provided a rationale for the extra term, showing it to be a Wess-Zumino (WZ) term in the sense that it could be obtained by applying to the global supersymmetry (SUSY) group the construction that yields the WZ term for sigma models on ordinary group manifolds [2].

The identification of a WZ term in the superstring action raises further questions. Normally such a term is expected only when the group manifold is topologically nontrivial, specifically when the third homology group (for a two-dimensional field theory) does not vanish. There is then a topological quantization

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