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Dilations of Quantum Dynamical Semigroups with Classical Brownian Motion

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Abstract. We show that any quantum dynamical semigroup can be written with the help of the solution of a vector-valued classical stochastic differential equation. Moreover this equation leads to a natural construction of a unitary dilation in term of Wiener spaces.

1. Introduction

In this paper we study dilations of quantum dynamical semigroups. Such semigroups are the quantum analogues of classical Markov evolutions and they describe irreversibility without memory effects. The mathematical structure of quantum dynamical semigroups and their generators has been studied by many authors e.g. [1–4]. The dilation problem consists now in constructing a larger reversible dynamical system in such a way that the semigroup evolution arises as a reduced dynamics. The existence of dilations of quantum dynamical semigroups has been studied from a purely mathematical point of view in [5]. Recently, however, quite a few papers have been published in order to obtain explicit and physically relevant constructions of such dilations [6–8].

It is well known that classical Markov evolutions are generated by stochastic equations of Langevin type driven by Brownian motions. For quantum dynamical semigroups a construction of the following nature has been described by [9–11]. The quantum system is coupled to either Fermi or Bose fields of dimension 1 (both cases can be handled). Then a reversible dynamics of the composed system is constructed with the help of a quantum stochastic differential equation in the sense that not only the equation is operator-valued but also the noise terms are given by quantum fields. The fact that the noise terms do not commute is essential in this scheme in order to obtain a general quantum dynamical semigroup as reduced dynamics. Technically this approach uses therefore a non-commutative version of the Ito stochastic calculus.

Our approach is quite different [12]. We start from the observation that the

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