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A Lipatov Bound for Φ_4^4 Euclidean Field Theory

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Abstract. We bound the large order behavior of these pieces of the renormalized perturbation expansion for Φ_4^4 which do not contain "renormalons" [1]. The bound we obtain has the form of the leading asymptotic behavior computed by the Lipatov method, with the exact value of the "Lipatov constant." Therefore, this paper is a step towards the rigorous study of the large order behavior of Φ_4^4 and towards a proof of existence of the first "renormalon" singularity which should prevent the theory from being Borel summable. Using the results of this paper and the techniques of [15], one can for instance improve [17] the estimate [18] on the finiteness of the radius of convergence of the Borel transform of renormalized Φ_4^4 and obtain that this radius is at least the exact value conjectured in [1].

I. Introduction

In this paper we prove an upper bound of the "Lipatov" type which applies in particular to the convergent graphs of Φ_4^4. We hope that this partial result will be relevant for a more complete future study of the large order behavior of Φ_4^4. This large order behavior is expected to be governed by the presence of a renormalon singularity [1] on the positive real axis of the Borel transform. It happens that this singularity is indeed closer to the origin than the "instanton" singularity on the negative real axis which is responsible for the "Lipatov" behavior of the Φ^4 theory at large order in lower dimensions [2, 3] (see Fig. 1). In the lower dimensions (1, 2, and 3) where the theory is superrenormalizable, the rigorous analysis of the leading behavior of the perturbative expansion has been completed [4–8]. Therefore, we think that the next important objective in this domain should be to find this leading behavior for the renormalized perturbation expansion of Φ_4^4, and more

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