Commun. Math. Phys. 108, 153–175 (1987)

Communications in Mathematical Physics

Hyperbolic Conservation Laws with Relaxation*

Tai-Ping Liu

Department of Mathematics, University of Maryland, College Park, MD 20742, USA

Abstract. The effect of relaxation is important in many physical situations. It is present in the kinetic theory of gases, elasticity with memory, gas flow with thermo-non-equilibrium, water waves, etc. The governing equations often take the form of hyperbolic conservation laws with lower-order terms. In this article, we present and analyze a simple model of hyperbolic conservation laws with relaxation effects. Dynamic subcharacteristics governing the propagation of disturbances over strong wave forms are identified. Stability criteria for diffusion waves, expansion waves and traveling waves are found and justified nonlinearly. Time-asymptotic expansion and the energy method are used in the analysis. For dissipative waves, the expansion is similar in spirit to the Chapman-Enskog expansion in the kinetic theory. For shock waves, however, a different approach is needed.

1. Introduction

The phenomenon of relaxation is important in many physical situations. In the kinetic theory of monatomic gases, when an equilibrium state is perturbed, it gradually relaxes to the equilibrium state with Maxwellian velocity distribution. In the continuum theory of nonmonatomic gases, there are other modes of internal energy besides the translated one, and when the gas is perturbed, the translational energy adjusts to its equilibrium value quickly. Other modes relax to their equilibrium values through collision of gas particles. The time scale for such a relaxation process may not be short and the phenomenon of thermonon-equilibrium becomes important. In this case, the compressible Euler equations should be supplemented by a rate equation governing the nonequilibrium mode of the internal energy. For elastic material with memory, the stress depends on the past history of the strain, and a perturbation of a constant state relaxes to the state satisfying the equilibrium elastic stress-strain relation. For a river flow to be in

* The paper was written at Mittag-Leffler Institute; the author wants to thank the Institute for the visiting position in 1986. This work was supported in part by an NSF grant