

Topological Anomalies: Explicit Examples

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Abstract. We discuss the mathematical picture of anomalies. By solving the Dirac equation in the background of non-trivial families of gauge connections, we show explicitly the interplay between spectral flows, zero modes of the Dirac operator and projective representations of the gauge group, and the existence of both perturbative and non-perturbative anomalies. We give an explicit expression for the fermion determinant for chiral QCD in two dimensions when an anomaly is present.

1. Introduction

In recent years, considerable progress has been made in understanding the structure of the anomalies and their topological origin. By exploiting the cohomological properties [1, 2] of the Wess-Zumino consistent condition, one can derive the descent equations, and using these, a straightforward way of writing down gauge anomalies can be prescribed. Finally, one can give topological interpretations to these essentially algebraic constructions [3]. The mathematical foundations for anomalies were reviewed by Atiyah and Singer, and discussed within the framework of the family index theorem of elliptic operators [4].

However, the physical implications of the anomalies remain largely to be explored. As is well known, anomalies pose serious problems for the quantization of chiral fermions in gauge theories. In building models of elementary particles, the current wisdom is that anomalies are inconsistencies in the theory and have to be cancelled, giving rise to severe constraints on the resultant particle content. What other kinds of constraints do anomalies imply? It is, of course, far from clear that current methods of handling quantum field theories are adequate in the presence of anomalies, and if not, the structure of the anomalies may help in pointing out alternatives. In addition, there is also the question that even if anomalies are cancelled, what role each individual anomaly plays in the theory, especially in models of composite fermions. To further explore these questions, it is useful to examine more closely the structure of anomalies in the form of explicit solutions in