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A Generalization of the Momentum Mapping Construction for Quaternionic Kähler Manifolds

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Abstract. We present a method of reduction of any quaternionic Kähler manifold with isometries to another quaternionic Kähler manifold in which the isometries are divided out. Our method is a generalization of the Marsden-Weinstein construction for symplectic manifolds to the non-symplectic geometry of the quaternionic Kähler case. We compare our results with the known construction for Kähler and hyperKähler manifolds. We also discuss the relevance of our results to the physics of supersymmetric non-linear σ-models and some applications of the method. In particular, we show that the Wolf spaces can be obtained as the $U(1)$ and $SU(2)$ quotients of quaternionic projective space $\mathbb{H}P(n)$. We also construct an interesting example of compact riemannian V-manifolds *(orbifolds)* whose metrics are quaternionic Kähler and not symmetric.

1. Introduction

Quaternionic Kähler and hyperKähler manifolds are of increasing interest to both physicists and mathematicians. In quantum field theory nonlinear σ-model lagrangians with self-interacting scalar fields on these manifolds play a very special rôle: they admit supersymmetric extensions. It is very well known that in 4-dimensional spacetime $N=1$ ($N=2$) globally supersymmetric interactions of bosons and fermions are determined by geometry of a Kähler (hyperKähler) manifold M [1, 2]. Scalar σ-model fields $\phi(x)$ are then maps from 4-dimensional coordinate space (for instance Minkowski or Euclidean space) into M. In $N=2$ local supersymmetry the situation is different: The riemannian manifold M is restricted to be quaternionic Kähler manifold of negative scalar curvature [3].

A more realistic picture must include also interacting gauge bosons. Thus, one would like to be able to couple fermionic and bosonic σ-model matter fields to the Yang-Mills vector multiplet without breaking supersymmetry. This issue was first

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