Commun. Math. Phys. 108, 91–115 (1987)

Communications in Mathematical Physics

Generally Covariant Quantum Field Theory and Scaling Limits

Klaus Fredenhagen and Rudolf Haag

II. Institut für Theoretische Physik der Universität Hamburg, Luruper Chaussee 149, D-2000 Hamburg 50, Federal Republic of Germany

Abstract. The formulation of a generally covariant quantum field theory is described. It demands the elimination of global features and a characterization of the theory in terms of the allowed germs of families of states. A simple application is the computation of counting rates of accelerated idealized detectors. As a first orientation we discuss here the consequences of the assumption that the states have a short distance scaling limit. The scaling limit at a point gives a reduction of the theory to tangent space. It contains kinematical information but not the full dynamical laws. The reduced theory will, under rather general conditions, be invariant under translations and under a proper subgroup of the linear transformations in tangent space. One interesting possibility is that it is invariant under SLR(4). Then the macroscopic metric must evolve as a cooperative effect in finite size regions. The other natural possibility is that each family (coherent folium) of states defines a microscopic metric by the scaling limit and the tangent space theory reduces to a theory of free massless fields in a Minkowski space. Irrespective of the assumption of a scaling limit we show that the theory can be constructed from strictly local information.

I. The Physical Picture

Quantum Field Theory may be regarded as a synthesis of special relativity (incorporating the restricted principle of locality) and quantum theory. In classical physics the principle of locality, originating in the development of electro-dynamics by Faraday and Maxwell, was sharpened in special relativity theory by the statement that "no causal influence can propagate faster than light" and was ultimately implemented in the general theory of relativity in its most stringent form: *The laws of nature regulate only the behavior of physical quantities in the infinitesimal neighborhood of each point.*

In special relativity there is still one last remnant of global laws, namely the rigid metric structure of space-time. This is removed in the general relativity theory in the following way:

(i) The basic physical quantities relate to the tangent spaces at the points of the 4-