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Anderson Localization for Bernoulli and Other Singular Potentials

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Abstract. We prove exponential localization in the Anderson model under very weak assumptions on the potential distribution. In one dimension we allow any measure which is not concentrated on a single point and possesses some finite moment. In particular this solves the longstanding problem of localization for Bernoulli potentials (i.e., potentials that take only two values). In dimensions greater than one we prove localization at high disorder for potentials with Hölder continuous distributions and for bounded potentials whose distribution is a convex combination of a Hölder continuous distribution with high disorder and an arbitrary distribution. These include potentials with singular distributions.

We also show that for certain Bernoulli potentials in one dimension the integrated density of states has a nontrivial singular component.

1. Introduction

The Anderson tight-binding model is given by the random Hamiltonian $H = -\Delta + v$ on $l^2(\mathbb{Z}^\nu)$, where $(-\Delta)(x, y) = -1$ if $|x-y| = 1$ and zero otherwise, and $v(x)$, $x \in \mathbb{Z}^\nu$, are independent identically distributed random variables with common probability distribution μ.

In one dimension the spectral properties of this random Hamiltonian have been fairly well understood for absolutely continuous μ with a "nice" density (see for example the year old reviews [1, 2]). In higher dimensions the first results towards localization were due to Fröhlich and Spencer [3]. These were followed by a proof of localization for a hierarchical version of H by Jona–Lasinio, Martinelli and Scoppola [4] based on the methods of [3], and by a short proof of the absence of absolutely continuous spectrum for Anderson's model at high disorder or low energy given by Martinelli and Scoppola [5], again based on [3]. Subsequently proofs of localization for the multidimensional Anderson model in the same region

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