Commun. Math. Phys. 108, 1–12 (1987)

Communications in Mathematical Physics

Geometric Expansion of the Boundary Free Energy of a Dilute Gas

Pierre Collet and François Dunlop

Centre de Physique Théorique de l'Ecole Polytechnique, Plateau de Palaiseau, F-91128 Palaiseau Cedex, France

Abstract. We consider a dilute classical gas in a volume $\varepsilon^{-1}\Lambda$ which tends to $\mathbb{R}^d$ by dilation as $\varepsilon \to 0$. We prove that the pressure $p(\varepsilon^{-1}\Lambda)$ is C^q in ε at $\varepsilon = 0$ (thermodynamic limit), for any $q \in \mathbb{N}$, provided the boundary $\partial\Lambda$ is C^q and provided the Ursell functions $u_n(x_1, \ldots, x_n)$ admit moments of degree q and have "nice" derivatives.

1. Introduction

In a recent paper [1], Pogosian derives the asymptotic expansion of the pressure $p(\varepsilon^{-1}\Lambda)$ in the thermodynamic limit $\varepsilon \to 0$, up to order d in ε,

$$p(\varepsilon^{-1}\Lambda) = a_0(\Lambda) + a_1(\Lambda)\varepsilon + \cdots + a_{d-1}(\Lambda)\varepsilon^{d-1} + a_d(\Lambda)\varepsilon^d + r_d(\varepsilon, \Lambda)\varepsilon^d$$

for a dilute gas in $\Lambda \subset \mathbb{Z}^d$ or $\Lambda \subset \mathbb{R}^d$. The remainder satisfies

$$|r_d(\varepsilon, \Lambda)| \leqq \begin{cases} 0(\log \varepsilon^{-1}) & \text{in general} \\ 0(1) & d = 2 \\ 0(\varepsilon) & \text{if } \Lambda \subset \mathbb{Z}^d \text{ or } \Lambda \text{ polyhedron in } \mathbb{R}^d. \end{cases}$$

The hypotheses on $\partial\Lambda$ are the natural ones, the hypotheses on the interaction potential are rather complicated and are not optimal. The proof is based on the Mayer expansion and extensive use of Taylor expansions.

The present paper extends the above results and simplifies the proofs, for volumes $\varepsilon^{-1}\Lambda \subset \mathbb{R}^d$ with $\partial\Lambda$ smooth. We prove the absence of logarithms (as conjectured by Pogosian), and extend the expansion to all orders. The order d (dimension of space) has nothing special to it when the interaction is smooth, which we assume, as Pogosian does in his proof if we understand it correctly. It is clear however that strong singularities in the interaction potential would show up in the expansion at some order in ε depending on the dimension. We do not know whether a jump discontinuity in the Ursell functions (e.g. square hard core potential) would spoil the expansion at all.