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On the Existence of n-Geodesically Complete or Future Complete Solutions of Einstein's Field Equations with Smooth Asymptotic Structure

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Abstract. It is demonstrated that initial data sufficiently close to De-Sitter data develop into solutions of Einstein's equations $\mathrm{Ric}[g]=\Lambda g$ with positive cosmological constant Λ, which are asymptotically simple in the past as well as in the future, whence null geodesically complete. Furthermore it is shown that hyperboloidal initial data (describing hypersurfaces which intersect future null infinity in a space-like two-sphere), which are sufficiently close to Minkowskian hyperboloidal data, develop into future asymptotically simple whence null geodesically future complete solutions of Einstein's equations $\mathrm{Ric}[g]=0$, for which future null infinity forms a regular cone with vertex i^+ that represents future time-like infinity.

1. Introduction

In this paper previous investigations [6, 9] of the existence of asymptotically simple solutions of Einstein's equations $\mathrm{Ric}[\tilde{g}]=\Lambda\tilde{g}$ with cosmological constant $\Lambda \geqslant 0$ [$\mathrm{sign}=(-,+,+,+)$] will be extended. We will first discuss the case of positive cosmological constant, since there the results are of a certain completeness now.

In [9] the constraint equations implied by the "regular conformal field equations" [4] on past conformal infinity $\mathscr{I}^-$ have been analysed, which in the case $\Lambda>0$ is space-like. It turned out that there is no need to consider an analogue of the Lichnerowicz equation if one wants to provide solutions of the constraint equations. Let S be an arbitrary orientable compact 3-dimensional manifold endowed with a Riemannian metric $h_{\alpha\beta}$, Λ a positive number, and $d_{\alpha\beta}$ a symmetric trace-free tensor field on S satisfying the equation $D^\alpha d_{\alpha\beta}=0$, where D denotes the covariant Levi-Civita derivative operator for $h_{\alpha\beta}$. Then from these fields a complete "asymptotic initial data set" for the regular conformal field equations can be derived by differentiation and algebra such that S together with these data describes the geometry of a past conformal infinity $\mathscr{I}^-$. Moreover, all these (sufficiently smooth) initial data sets determine unique past asymptotically simple