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Many Phases in Systems without Periodic Ground States

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Abstract. The low temperature behavior of systems without periodic ground states is investigated. It is shown by using Peierls' argument that in some models the translational symmetry is broken. In particular, an infinite range model with infinitely many Gibbs states is constructed.

1. Introduction

One of the unsolved problems of statistical mechanics is to understand the causes of crystalline symmetry in low temperature matter [1]. Here we discuss specific classical lattice gas models in two dimensions. Namely, each site of the square lattice can be occupied by one of several different particles. The particles interact through two-body potentials and low temperature behavior of such systems results from the competition between energy and entropy, i.e., the minimization of the free energy.

Before discussing the finite temperature region it is important to investigate the ground states of the system, i.e., the configurations of particles with minimal potential energy per lattice site. A major question here is: does every classical lattice system always possess at least one periodic ground state? This problem was studied by Radin [2–4] and the answer is no. There are examples of models which do not have periodic ground states. They are based on Berger's [5] and Robinson's [6] results on nonperiodic tilings of the plane with a finite family of certain tiles. Such models do possess, however, so-called q-periodic ground states. A configuration of particles is q-periodic if when a certain fraction of them is ignored the rest of the configuration is periodic; the smaller the fraction, the larger the period.

The next step is to prove or disprove that in such systems the q-periodic structure can survive at low temperatures giving rise to a q-periodic Gibbs state. We have to keep in mind, however, that in contradistinction to the ground states there is always at least one translational invariant Gibbs state at any nonzero temperature. We have not resolved this problem. Instead we answer here the