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A Conformal Holomorphic Field Theory

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Abstract. A formulation of a field theory on the complex Minkowski space in terms of complex differential geometry is proposed. It is also shown that our model of field theory differs from the standard model on the real Minkowski space only in the limit of high energy.

1. Introduction

The aim of this paper is the construction of field theory for the massive conformal particle interacting with an external field. There are important reasons, having their roots in conformal symplectic geometry to investigate such a theory on the complex Minkowski space. We believe that the utilization of the complex Minkowski space as a base for construction of the field theory is not only useful technically (see e.g. [13]), but can be of great physical importance.

In the following, by a scalar massive conformal particle we will understand a physical object localized in time and space with a given energy and momentum. The conformal particle mass may change when the particle interacts with a field, contrary to the relativistic particle mass. Due to this fact the phase space of a scalar massive conformal particle is an eight – dimensional conformal Hamiltonian space (see [7, 10]). Considering also scalar conformal anti-particles, scalar conformal tachyons and scalar massless conformal particles we find (see Sect. 2) that two models of kinematics of conformal scalar particles exist. The phase space of the first model (which will be called the nilpotent model) is the cotangent bundle T^*M of the conformal compactification M of Minkowski space. In the second model (holomorphic one) the phase space is given as the complexification $\mathbb{M}$ of M. It turns out that the conformally invariant symplectic form ω_h on $\mathbb{M}$ depends on the real parameter h and $(\mathbb{M}, \omega_h)$ corresponds to (T^*M, ω_0) when $h \to 0$, where ω_0 stands for a canonical symplectic form on T^*M. In other words the nilpotent model is the limiting case of the holomorphic one.

Taking into account the above facts we construct field theory on the complex Minkowski space (the configuration space of holomorphic kinematics) which, at