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On the Continuous Limit for a System of Classical Spins

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Abstract. The continuum limit of a cubic lattice of classical spins processing in the magnetic field created by their closest neighbours is considered. Results concerning existence, uniqueness and (for initially small spin deviation) long time behaviour, are presented.

1. Introduction

A classical model for an isotropic ferromagnet is provided by a collection of three-dimensional spin vectors with unit length and arbitrary directions, located at the nodes of a d-dimensional cubic lattice. We denote by S_i or $S(x_i)$ the (classical) spin located at the point $x_i = n_{i_1}h_1 + \ldots + n_{i_d}h_d$, where the n_{i_j}'s are integers, and h_j the mesh vector in the j-direction. We assume that all the mesh vectors have the same length h.

Concerning the dynamics, a simple hypothesis consists in assuming that each spin $S(x_i)$ processes in the local magnetic field $\sum_{j=1}^{d} S(x_i + h_j) + S(x_i - h_j)$ created by the closest neighbours. The equations of motion are written [1, 3, 10, 11]

$$\frac{dS}{dt^*}(x_i) = J \sum_{j=1}^{d} S(x_i) \wedge (S(x_i + h_j) + S(x_i - h_j)), \tag{1.1}$$

where $\wedge$ denotes vectorial product and J the (positive) nearest neighbour exchange coupling constant. Equation (1.1) can be rewritten:

$$\frac{dS}{dt}(x_i) = \sum_{j=1}^{d} S(x_i) \wedge \frac{S(x_i + h_j) - 2S(x_i) + S(x_i - h_j)}{h^2} \tag{1.2}$$

with

$$t = h^2 t^*/J .$$

When interested in phenomena at scales large compared to the lattice mesh size and with time scale $O(1/h^2)$, we are led to consider the limit $h \to 0$ in Eq. (1.2).