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The Gibbs Measures and Partial Differential Equations

I. Ideas and Local Aspects

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Abstract. We investigate the connections of the Gibbs measures, which appear in Euclidean Field Theory, and the corresponding partial differential equations of Classical Euclidean Field Theory.

1. Preliminaries

Let $\mathscr{F}$ denote the family of bounded open sets $\Lambda \subset R^d$ with piecewise $\mathscr{C}^n$, for some $n \geqq 1$, boundary $\partial\Lambda$. Let $\mathscr{F}_0$ be a countable base of $\mathscr{F}$, i.e.

$$\mathscr{F}_0 := \{\Lambda_n\}_{n\in\mathbb{N}} \subset \mathscr{F} \quad \text{such that for every} \quad n \in \mathbb{N}, \Lambda_n \subset \Lambda_{n+1} \quad \text{and} \quad \forall \Lambda \in \mathscr{F}\ \exists n\ \Lambda \subset \Lambda_n . \tag{1}$$

For $\Lambda \subset R^d$ we denote $\Lambda^c := R^d \backslash \Lambda$.

Let (Ω, Σ) be a standard Borel space. We assume that in Σ there is a distinguished family of σ-algebras of local events $\{\Sigma_\Lambda\}_{\Lambda\in\mathscr{F}}$, which generates Σ and is compatible, i.e.

$$\Lambda_1, \Lambda_2 \in \mathscr{F} : \Lambda_1 \subset \Lambda_2 \ \Rightarrow\ \Sigma_{\Lambda_1} \subset \Sigma_{\Lambda_2} . \tag{2}$$

For any open set $Q \subset R^d$ we define the σ-algebra Σ_Q as the σ-algebra generated by $\{\Sigma_\Lambda : \Lambda \in \mathscr{F}, \Lambda \subset Q\}$. For arbitrary set $Q \subset R^d$ we define

$$\Sigma_Q := \{\bigcap \Sigma_{\tilde{Q}} : \tilde{Q} \text{ open}, \tilde{Q} \supset Q\} . \tag{3}$$

In particular we have the family of σ-algebras $\{\Sigma_{\Lambda^c}\}_{\Lambda\in\mathscr{F}}$ with the property

$$\Lambda_1, \Lambda_2 \in \mathscr{F} : \Lambda_1 \subset \Lambda_2 \ \Rightarrow\ \Sigma_{\Lambda_2^c} \subset \Sigma_{\Lambda_1^c} . \tag{4}$$

We define the σ-algebra at infinity by

$$\Sigma_\infty := \bigcap_{\Lambda\in\mathscr{F}} \Sigma_{\Lambda^c} . \tag{5}$$

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