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Invariant Circles for the Piecewise Linear Standard Map

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Abstract. We investigate invariant circles for a one-parameter family of piecewise linear twist homeomorphisms of the annulus. We show that invariant circles of all types and rotation numbers occur and we classify them into families. We compute parameter ranges in which there are no invariant circles.

1. Introduction

We investigate invariant circles for the one-parameter family $h_k (k \in \mathbb{R})$ of homeomorphisms of the annulus $S^1 \times \mathbb{R}$ defined by

$$h_k(x, y) = (x + y + kg(x), y + kg(x)), \qquad (*)$$

where $g: S^1 \to \mathbb{R}$ is the piecewise linear function $g(x) = |x - 1/2| - 1/4$, and S^1 is parametrised as $\mathbb{R}/\mathbb{Z}$.

We call h_k the *piecewise linear standard map* since it is obtained from the *standard map*

$$s_k(x, y) = \left(x + y + \frac{k}{4}\cos 2\pi x, y + \frac{k}{4}\cos 2\pi x \right)$$

by replacing $\cos 2\pi x$ by its crudest piecewise linear approximation.

For any continuous function g the homeomorphism h_k defined by $(*)$ satisfies the *twist condition*, that is to say, if $\tilde{h}_k$ denotes the lift of h_k to the universal cover $\mathbb{R} \times \mathbb{R}$ of the annulus and p_1 denotes the projection of $\mathbb{R} \times \mathbb{R}$ onto its first factor, then

$$p_1 \tilde{h}_k(x, y_2) > p_1 \tilde{h}_k(x, y_1) \quad \text{whenever } y_2 > y_1 .$$

Furthermore such an h_k preserves area, and if

$$\int_0^1 g(x)dx = 0$$