Commun. Math. Phys. 107, 233–240 (1986)

Communications in
Mathematical
Physics

On the Local Implementations of Gauge Symmetries in Local Quantum Theory

Francesco Fidaleo
Dipartimento di Matematica, Università di Roma "La Sapienza", I-00185 Roma, Italy

Abstract. Under the general assumptions of quantum field theory in terms of local algebras of field operators fulfilling the split property, we prove that any two local covariant implementations of the gauge group (or, in the case of a connected and simply connected Lie gauge group, any two choices of local current algebras) relative to a pair of double cones $\mathcal{O}_1, \mathcal{O}_2$, are related by a unitary equivalence induced by a unitary in the algebra of observables localized in $\tilde{\mathcal{O}}_2$ which commutes with all fields localized in $\tilde{\mathcal{O}}_1$, where $\tilde{\mathcal{O}}_1$ is any double cone contained in the interior of $\mathcal{O}_1$, and $\tilde{\mathcal{O}}_2$ any double cone containing $\mathcal{O}_2$ in its interior.

1. Introduction

Recently the possibility of implementing locally the symmetries of a local quantum theory has been studied ([1–3]). One of the main motivations was to give a quantum version of classical Noether's theorem.

In [1, 2] a sufficient condition, the split property, is given for the local implementability of gauge transformations. Under this hypothesis, all the other symmetries which are present in the theory can be locally implemented as well (e.g. space-time symmetries and supersymmetric transformations; see [3]). The split property can be grounded on general properties of quantum field theory; see [4, 3].

A local implementation of the gauge transformations is a representation of the gauge group in a local field algebra, say $F(\mathcal{O}_2)$, which induces the gauge transformations on the field algebra $F(\mathcal{O}_1)$ associated to a "smaller" region $\mathcal{O}_1$.

The aim of this paper is to prove the next result. Two local implementations for the regions $\mathcal{O}_1, \mathcal{O}_2$, will be equivalent by a "well localized" unitary in the observable algebra.

To simplify, we deal with theories with only localizable charges [16]. We suppose in fact that we have given local net of fields on a Hilbert space $\mathcal{H}$ and a gauge group G representated on $\mathcal{H}$. The local fields generate a field algebra $\mathcal{F}$ and the observable algebra $\mathcal{A}$ is derived from $\mathcal{F}$ by the principle of gauge invariance.