

The Analysis of Elliptic Families

II. Dirac Operators, Êta Invariants, and the Holonomy Theorem

Jean-Michel Bismut¹ and Daniel S. Freed²

¹ Université Paris-Sud, Département de Mathématique, Bâtiment 425, F-91405 Orsay, France

² M.I.T. Department of Mathematics, Cambridge, MA 02139, USA

Abstract. In this paper we specialize the results obtained in [BF 1] to the case of a family of Dirac operators. We first calculate the curvature of the unitary connection on the determinant bundle which we introduced in [BF 1].

We also calculate the odd Chern forms of Quillen for a family of self-adjoint Dirac operators and give a simple proof of certain results of Atiyah-Patodi-Singer on êta invariants.

We finally give a heat equation proof of the holonomy theorem, in the form suggested by Witten [W 1, 2].

Contents

Introduction	104
I. A connection on the Determinant Bundle of a Family of Dirac Operators	106
a) Clifford Algebras: The Even Dimensional Case	106
b) Clifford Algebras: The Odd Dimensional Case.	107
c) Description of the Fibered Manifold	107
d) Connections on TM	108
e) Connections on Infinite Dimensional Bundles	109
f) A Connection on the Determinant Bundle of a Family of First Order Differential Operators	110
g) The Case of a Family of Dirac Operators: Explicit Computation of the Curvature of the Determinant Bundle	113
II. Dirac Operators on Odd Dimensional Manifolds.	117
a) Assumptions and Notations.	117
b) An Auxiliary Grassmann Variable	118
c) The Asymptotics of Certain Heat Kernels.	118
d) Local Regularity of the Êta Invariant	120
e) The Variation of the Êta Invariant.	121
f) Odd Chern Forms, Êta Invariant and the Spectral Flow	122
III. The Holonomy Theorem: A Heat Equation Proof	128
a) Assumptions and Notations.	129
b) A Generalized Lichnerowicz Formula	129
c) Construction of Certain Heat Kernels	130
d) Local Cancellation Properties of the Connection ${}^1\nabla$	131