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The Motion of Kaluza-Klein Monopoles

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Abstract. A scheme is proposed and justified for examining the motion of the five dimensional Kaluza-Klein monopoles at low energy. The classical and quantum scattering is discussed and it is shown that for all separations and at small velocities the monopoles do not interact with one another.

1. Introduction

There has recently been progress in discovering the approximate motion of solitons arising in field theories, Yang-Mills-Higgs theory [1] and the $\mathbb{C}P^1$ model [2]. This is of interest for two reasons; first, if one adopts the view that solitons may correspond to physical particles, then one has information on their dynamics and their interactions. Secondly, in the light of the conjectured dualities between solitons as non-perturbative phenomena and perturbative quanta in field theories, such information would either corroborate or alternatively completely eliminate the possibility of such a duality. These recent results have been obtained by constructing approximate time independent solutions of the field equations, which are successions of exact static solutions, the quasistatic approximation. The motion is geodesic on the space of static N-soliton solutions equipped with a natural metric.

The aim of this article is to extend this scheme to the solitons of a gravity theory – five dimensional Kaluza-Klein theory [3] – and is organised as follows. In Sect. 2 the monopoles of Gross, Perry, and Sorkin [4] are described. In Sect. 3 the space of gravitational degrees of freedom is discussed and the moduli space of N-monopole metrics is obtained. In Sect. 4 the approximation of geodesic motion on the moduli space is justified and in Sect. 5 the metric is constructed. Section 6 discusses the application of the metric to the problem of classical and quantum scattering and the accuracy of the scheme is considered. Section 7 concludes the article.

2. Geometry of the Kaluza-Klein Monopoles

Simple Kaluza-Klein theory is five dimensional General Relativity with the restriction that the fith dimension is given by a circle. That is, the coordinate x^5 is