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Asymptotic Completeness for a New Class of Stark Effect Hamiltonians

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Abstract. Existence and completeness of the wave operators is shown for the Stark effect Hamiltonian in one dimension with a potential $V = W''$, where W is a bounded function with four bounded derivatives. This class of potentials include some almost periodic functions and periodic functions with average zero over a period (Stark–Wannier Hamiltonians). In the last section we discuss classical particle scattering for the same class of potentials.

1. Introduction. Statement of Result

In this paper unitary equivalence of a class of one-dimensional Stark effect Hamiltonians with bounded potentials is shown. The new results are that no decay of the potential in the direction of the field is required and that the wave operators exist and are complete.

Let $H_0 = -d^2/dx^2 + x$ denote the free Stark effect Hamiltonian in $L^2(\mathbb{R})$. Let $B^4(\mathbb{R})$ denote the four times differentiable functions on $\mathbb{R}$ which are bounded with all derivatives bounded. The main result of this paper is

Theorem 1.1. *Let $V = W''$, $W \in B^4(\mathbb{R})$, a real-valued function, and let $H = H_0 + V$. Then the wave operators*

$$W_{\pm} = \lim_{t \to \pm\infty} e^{itH} e^{-itH_0}$$

exist and are unitary.

The wave operators give a unitary equivalence between H and H_0. H_0 has purely absolutely continuous spectrum equal to $\mathbb{R}$, hence the same holds for H. Absence of singular continuous spectrum has been shown for a larger class of potentials in [3, 4, 11]. Absence of eigenvalues is a classical result from the theory of ordinary differential equations, see e.g. [5].

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