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Large-Time Behavior of Discrete Velocity Boltzmann Equations

J. Thomas Beale[1]

Department of Mathematics, Duke University, Durham, NC 27706, USA

Abstract. We study the asymptotic behavior of equations representing one-dimensional motions in a fictitious gas with a discrete set of velocities. The solutions considered have finite mass but arbitrary amplitude. With certain assumptions, every solution approaches a state in which each component is a traveling wave without interaction. The techniques are similar to those in an earlier treatment of the Broadwell model [1].

1. Introduction

The purpose of this paper is to describe the evolution at large times of solutions to certain systems of semilinear hyperbolic equations. These equations represent one-dimensional motions in a fictitious gas consisting of particles with a discrete set of possible velocities. The study of such model gases began with Maxwell. The systems studied here have the form

$$u_{i,t}+c_i u_{i,x}=\sum_{j,k=1}^{n} a_i^{jk} u_j u_k \equiv F_i, \qquad i=1,\dots,n, \tag{1.1}$$

where $u_i(x,t)$ is the density of particles with speed c_i, and the interaction coefficients a_i^{jk}, as well as the speeds, are constant. We suppose that the speeds are distinct:

$$c_i \neq c_j \quad \text{if} \quad i \neq j. \tag{1.2}$$

With certain assumptions on the system, we show that each solution of finite total mass, but arbitrary amplitude, evolves toward a state in which the interaction is negligible; i.e., there are limiting wave forms u_i^∞ so that

$$u_i(x,t) \sim u_i^\infty(x-c_i t) \quad \text{as} \quad t\to\infty, \qquad i=1,\dots,n.$$

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