Commun. Math. Phys. 106, 551–568 (1986)

Communications in
Mathematical
Physics

On Local Solutions of the Initial Value Problem for the Vlasov–Maxwell Equation

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Abstract. The initial value problem of the Vlasov–Maxwell equation has a unique solution in a time interval $[0, T]$ for each initial data in some function space. T is estimated by the size of the initial data. The solution is classical, if the initial data is smooth.

1. Introduction

The density distribution of the charged gas particles changes under the rule described as the Vlasov–Maxwell equation. In this paper we prove that the initial value problem for the Vlasov–Maxwell equation has a unique local (in time) solution for each initial data in a slightly wide class of functions.

Let $f_i = f_i(t, x, v)$ be the density distribution of the charged gas particles of the type i $(1 \leqq i \leqq N)$ at the time $t \geqq 0$ and the point $x \in R^3$ with the velocity $v \in R^3$. The Vlasov–Maxwell equation for $\{f_i\}$ is described in the following form:

$$\frac{\partial}{\partial t} f_i + v \cdot \nabla_x f_i + \frac{q_i}{m_i}\left(E + \frac{v}{c} \times B \right) \cdot \nabla_v f_i = 0 \quad (1 \leqq i \leqq N), \tag{1.1}$$

$$f_i|_{t=0} = f_{i,0}(x, v), \tag*{(1.1)$_0$}$$

$$\frac{\partial}{\partial t} E - c\nabla_x \times B = -4\pi \sum_{K=1}^{N} q_i \int v f_i(t, x, v) dv, \tag{1.2}$$

$$\frac{\partial}{\partial t} B + c\nabla_x \times E = 0.$$

$$E|_{t=0} = E_0(x), \quad B|_{t=0} = B_0(x), \tag*{(1.2)$_0$}$$

where E and B denote the electric and magnetic fields generated by the distributions f_i, m_i the mass and q_i the charge of the single particle of the i-species. The parameter $c \geqq 1$ denotes the light velocity. The notations $\cdot$ and $\times$ denote the scalar and vector products in R^3; $\nabla_x = {}^t(\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$ and $\nabla_v = {}^t(\partial/\partial v_1, \partial/\partial v_2, \partial/\partial v_3)$. Sometimes we use the notations $\langle , \rangle$ and $|\ \ |$ to denote the scalar product and the norm in R^n.