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A Non-Gaussian Renormalization Group Fixed Point for Hierarchical Scalar Lattice Field Theories

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Abstract. A rigorous method is developed to handle the "large field problems" in the Wilson-Kadanoff renormalization group approach to critical lattice systems of unbounded spins. We use this method to study in a hierarchical approximation the non-Gaussian renormalization group fixed point which governs the infrared behaviour of critical lattice field theories in three dimensions. The method is an improvement of the analyticity techniques of Gawedzki and Kupiainen: using Borel summation techniques we are able to incorporate the "large field region" into the "perturbative region" so that the theory is completely described in terms of convergent expansions.

1. Introduction

A major obstacle in the analysis of critical statistical mechanics systems is the lack of small expansion parameters. For example very little is known about scalar lattice spin systems with non-Gaussian critical long distance behavior

$$\int d\mu(\phi)\phi_i\phi_j \sim \frac{1}{|i-j|^{d-2+\eta}}, \qquad |i-j|\to\infty, \qquad \eta \geqq 0. \tag{1.1}$$

Here, $d\mu$ is a translation invariant Gibbs measure on configurations $\phi:\mathbb{Z}^d\to\mathbb{R}$. Although renormalization group (RG) concepts provide a powerful framework to think about such problems, it has been impossible so far to convert them into rigorous results, except for certain limit cases [10–12].

The methods presented in this paper are not restricted to such cases. However, we restrict our analysis to a class of hierarchical models. Such models have a long history in the testing of RG ideas [13–20, 25, 26]. We prove here the existence of a

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