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Asymptotics of the Cut Discontinuity and Large Order Behaviour from the Instanton Singularity: The Case of Lattice Schrödinger Operators with Exponential Disorder

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Abstract. We discuss the relation between the singularity structure of the Borel transform, the asymptotics of the cut discontinuity and the large order behaviour of perturbation theory. In an explicit example – a tight binding model with exponential disorder – we show how to obtain the first instanton singularity from a cluster expansion in the Borel variable, and as an application we determine the exact decay of the density of states as $E \to \infty$. The method opens some perspectives for similar problems arising from different models of Mathematical Physics.

1. Introduction

Perturbation expansions in Statistical Mechanics and QFT generally do not converge but are only asymptotic to the function under consideration. Even worse, asymptotic expansions do not determine their sum uniquely but there is a particularly convenient method invented by Borel, which under certain circumstances makes it possible to give an integral representation of the uniquely determined sum. The following result is well known.

Theorem 1. *(Watson-Nevanlinna). Let f be a function analytic in the half-plane*

$$D(R) = \{z \in \mathbb{C} : \operatorname{Re} z > R\}, \tag{1.1}$$

and have there an asymptotic expansion with remainder estimate (for arbitrary N)

$$\left| f(z) - \sum_{n=0}^{N-1} a_n z^{-n} \right| \leqq A\sigma^N N! |z|^{-N}. \tag{1.2}$$

Then the Borel transform

$$B(t) = \sum_{n=0}^{\infty} \frac{a_n}{n!} t^n \tag{1.3}$$

converges (at least) in the circle $\{t \in \mathbb{C} : |t| < \sigma^{-1}\}$ and has an analytic continuation to the region $S(\sigma) = \{t : \operatorname{dist}(t, R_+) < 1/\sigma\}$ satisfying the bound

$$|B(t)| \leqq \text{const} \exp(|t| \cdot R) \tag{1.4}$$