Commun. Math. Phys. 106, 459–466 (1986)

Communications in
Mathematical
Physics

Generic Triviality of Phase Diagrams in Spaces of Long-Range Interactions

Robert B. Israel*

Department of Mathematics, University of British Columbia, Vancouver, B.C., Canada, V6T 1Y4

Abstract. We show that interactions with multiple translation-invariant equilibrium states form a very "thin" set in spaces of long-range interactions of classical or quantum lattice systems. For example, generic finite-dimensional subspaces do not intersect this set. This constitutes a severe violation of the Gibbs Phase Rule.

1. Introduction

The work of Daniels and van Enter [1–3] has shown that phase transitions are less stable under long-range perturbations than had been believed, so that Ruelle's "heuristic theory of phase transitions" [7], and the strong form of the Gibbs Phase Rule that it implies, are violated in spaces of long-range interactions. In this paper we extend and generalize those results, and show that the instability of phase transitions is a generic phenomenon, in the sense of Baire category, in these spaces of interactions. This means that phase transitions occur only on a set which is in a sense very "thin," violating much weaker versions of the phase rule.

We will consider either a classical or quantum lattice system on $\mathbb{Z}^d$ (see [4] for notation). In the classical case the configuration space at each site is assumed to be a compact metric space. The Banach space $\mathscr{B}_g$ consists of those translation-invariant interactions Φ with

$$\|\Phi\|_g \equiv \sum_{0 \in X} g(|X|)\, \|\Phi(X)\| < \infty\,, \tag{1.1}$$

where g is some function on the positive integers, and $|X|$ is the cardinality of X. In order to define equilibrium states by the variational principle, we require $g(n) \geqq 1/n$ [in the case $g(n) = 1/n$ we will write $\mathscr{B}_g$ as $\mathscr{B}$]. Stronger conditions on g [e.g. $g(n) \geqq e^{an}$ for some $a > 0$] allow the use of DLR equations, KMS conditions, etc. Our results hold in any of these spaces, and thus are *not* connected to the

* Research supported by NSERC grant A-4015