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Convergence of Chorin-Marsden Product Formula in the Half-Plane

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Abstract. Consider a viscous incompressible fluid in the half-plane and let u_t be a solution of the Navier-Stokes equation. In this paper we prove that the product formula $(E_{t/n}G_{t/n}\phi u)^n u_0$, where E_t is the Euler flow, G_t is the heat flow and ϕ is a suitable operator describing the vorticity production due to the boundary, converges uniformly to u_t in the limit $n \to \infty$.

1. Introduction

The time evolution of a slightly viscous incompressible fluid in the presence of obstacles exhibits features which are difficult to investigate both from an analytical and a numerical point of view, even in the simplest two-dimensional case. In particular, large gradients of the velocity field, localized near the boundary, make difficult the use of the conventional algorithms, which are essentially based on projections on low frequency quantities.

To overcome this difficulty, Chorin [1] developed an algorithm which can be briefly described, as suggested by Marsden [5], in the following way. Denoting by E_t and G_t the Euler and the heat semiflows, respectively (G_t satisfying suitable boundary conditions), then an approximation at time t of the Navier-Stokes semiflow will be:

$$(E_{t/n}G_{t/n}\phi)^n, \tag{1.1}$$

where ϕ is a suitable operator describing the vorticity production due to the boundary and making the nonslip boundary conditions (in general destroyed by E_t and G_t) approximately satisfied.

The interest of the above method lies on the possibility of describing both E_t and G_t by means of particle dynamics (the particles are localized in points where the vorticity is sharply concentrated) thus taking into account, just from the very beginning, the high frequencies of the problem.

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