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The CP_{N-1} $1/N$-Action by Inverse Scattering Transformation in Angular Momentum

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Abstract. The effective action which generates $1/N$ expansion of the CP_{N-1} model in two dimensions is studied here by inverse-problem methods. The action contains a functional determinant, in which auxiliary scalar and vector fields are assumed to have a spherical symmetry. This leads to the introduction, as an associated linear problem, of a radial Schrödinger equation with two potentials v and θ, and a potential-dependent centrifugal term $\{(\ell - r\theta)^2/r^2 - 1/4r^2\}$. The full inverse scattering formalism is developed here for this diffusion problem. It is formulated in terms of two-component Jost solutions, and leads to a matricial Gel'fand-Levitan-Marchenko equation. The scattering data associated to the potentials by this IST are then used to obtain a closed local form for the whole effective action. This is indeed possible for the CP_{N-1} model, owing to the classical integrability. Moreover it is found that no spherically symmetric instanton exists in this case. However the absence of supplementary informations on the $1/N$ series, due to the non-integrability at quantum level, does not allow safe quantitative conclusions on the general behaviour of the $1/N$ series at large orders.

Introduction

The inverse problem methods have undergone important developments in the past years. They have been used to solve, first of all classically, then quantum-mechanically, a quantity of models in field theory and statistical mechanics. The central idea of those methods is the existence of a link (one-to-one correspondence) between a potential $v(x)$ and a set of scattering data, conveniently defined [1]. One can in this way solve non-linear equations that admit a Lax pair, i.e. a set of linear partial differential equations, the compatibility condition of which is the non-linear equation [2]. One of the linear differential equations is a diffusion problem, with the field(s) as potential; the other one indicates the time evolution of the

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