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The Dipole Phase in the Two-Dimensional Hierarchical Coulomb Gas: Analyticity and Correlations Decay

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Abstract. We illustrate the mechanism producing the dipole phase in a two dimensional Coulomb system by a detailed analysis of a hierarchical model. We prove the analyticity of the pressure and of the correlations for $\alpha^2 \equiv \beta e^2 > 8\pi$ (i.e. right above the usually conjectured value for the Kosterlitz-Thouless phase transition). We find also a power law decay for the correlations with exponent $\alpha^2/2\pi$ as the hierarchical distance goes to infinity.

1. Introduction

The renormalization group theory of the ultraviolet stability for scalar fields [1–3] hinted that the same techniques could be employed in the analysis of the two dimensional Coulomb gas of identical charges $\pm e$ even, and particularly, in the regime of the Kosterlitz-Thouless phase. The latter was rigorously established [4] for low temperatures, i.e. for $\beta e^2 \equiv \alpha^2 \gg 8\pi$, β being the inverse temperature and the Coulomb potential being normalized as

$$V(x-y) \simeq \frac{1}{2\pi} \log |x-y|^{-1} \quad \text{as} \quad |x-y| \to \infty \tag{1.1}$$

and regularized at short distance.

This paper is a continuation of the program started in [3] and continued in [5–7] to study the problem of the molecules formation in the two dimensional Coulomb gas and the related structure of the transition from the plasma phase to the dipole phase (Kosterlitz-Thouless transition).

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