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Analyticity of Correlation Functions for the Two-Dimensional Ising Model

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Abstract. Analyticity of correlation functions for the two-dimensional Ising model as a function of the inverse temperature except for the singularity at the critical temperature is proved. A crucial step is the establishment of the correspondence between extremal equilibrium states of the model and pure ground states of a one-dimensional spin system below the critical temperature T_c. An exact decay rate of the clustering property along axes is also determined for all $T \neq T_c$.

1. Main Results

We consider the two-dimensional Ising model with the Hamiltonian

$$H(\xi) = -\sum_{i,j} (J_1 \xi_{ij}\xi_{i+1,j} + J_2 \xi_{ij}\xi_{i,j+1}), \tag{1.1}$$

where $\xi_{ij} = \pm 1$, $(i,j) \in \mathbb{Z}^2$, and J's are real constants. We are interested in the thermodynamic limit $(L, M \to \infty)$

$$\psi_\beta(F) = \lim \langle F \rangle_{LM}, \qquad \langle F \rangle_{LM} = Z_{LM}^{-1} \sum_\xi F(\xi) e^{-\beta H^{LM}(\xi)}, \tag{1.2}$$

$$Z_{LM} = \sum_\xi \exp -\beta H^{LM}(\xi), \tag{1.3}$$

in which H^{LM} denotes (1.1) with the sum over ξ_{kl} with $(k,l) \in [-L, L] \times [-M, M]$ and we consider an arbitrary polynomial $F(\xi)$ of a finite number of ξ's, which we call a strictly local observable.

There is a critical inverse temperature β_c such that ψ_β is the unique equilibrium state for $|\beta| < \beta_c$ while there exist two extremal equilibrium states $\psi_{\beta\pm}$ with

$$\psi_\beta = (\psi_{\beta+} + \psi_{\beta-})/2 \tag{1.4}$$

for $|\beta| > \beta_c$ [1, 12]. Our main result is as follows: