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Constraints and Field Equations for Ten Dimensional Super Yang–Mills Theory†

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Abstract. We give a complete proof of the equivalence between constraint equations and field equations for the $d = 10$, $N = 1$ supersymmetric Yang–Mills theory, a result proposed and partially proved recently by Witten [1]. Our approach explicitly reconstructs the superconnection satisfying the constraints from the on shell component fields. A key ingredient of the method is the choice of a suitable family of gauges, effectively eliminating all gauge dependence on anti-commuting co-ordinates. As a corollary, obtained by dimensional reduction, we also deduce the equivalence of constraints and field equations for the $d = 4$, $N = 4$ theory, as well as for $d = 6$, $N = 2$.

1. Introduction

The purpose of the present work is to supplement some recent results of Witten's [1] concerning the relationship between the superconnection constraint equations and the supersymmetric Yang–Mills equations in ten dimensions. As pointed out in [1, 2], a natural set of constraint equations involves the vanishing of the supercurvature along super null lines. This gives rise to the super twistor correspondence, in which bundles $\tilde{E} \to \tilde{M}$ over super Minkowski space $\tilde{M}$, with a superconnection which is integrable along super null lines, correspond to certain bundles over the super ambitwistor space $\tilde{A}$, whose points are the super null lines in $\tilde{M}$. These bundles are characterized by the fact that they are trivial over certain quadrics $Q \subset \tilde{A}$ corresponding to the set of super null lines through the same point of $\tilde{M}$. The importance of such a construction lies in the fact that for a suitable choice of the superspace extension $\tilde{M}$ of the underlying Minkowski space M, the constraint equations, together with the Bianchi identities for the supercurvature, imply the

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