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Some Examples Concerning the Global Markov Property

Robert B. Israel*

Department of Mathematics, University of British Columbia, Vancouver, B.C., Canada V6T 1Y4

Abstract. We present examples of interactions of classical lattice systems whose extremal Gibbs states fail to have the global Markov property. One of the examples is translation invariant.

1. Introduction

Consider a classical lattice system on a lattice $\mathscr{L}$, with a finite-range interaction Φ $\left(\text{so that the formal Hamiltonian is } \sum_{X \subset \mathscr{L}} \Phi_X\right)$. Sites x, y will be said to be *neighbours* if $\Phi_X \neq 0$ for some finite $X \subset \mathscr{L}$ containing x and y. For any subset $\Lambda \subset \mathscr{L}$, let $\mathscr{F}_\Lambda$ be the σ-algebra generated by the spins in Λ, and let $\partial\Lambda$ be the set of sites not in Λ which have at least one neighbour in Λ. The Gibbs states for the interaction Φ satisfy the *local Markov property:*

$$E(f|\mathscr{F}_{\mathscr{L}\setminus\Lambda}) = E(f|\mathscr{F}_{\partial\Lambda}) \quad \text{for any finite } \Lambda \subset \mathscr{L} \text{ and bounded } \mathscr{F}_\Lambda\text{-measurable } f. \tag{1}$$

The *global Markov property* is the same statement, but with Λ allowed to be infinite.

It was conjectured in [2] that every extremal Gibbs state for Φ satisfies the global Markov property. A more general conjecture would be that for lattice random fields, the local Markov property plus triviality of the tail field imply the global Markov property. Von Weizsäcker [4] has found a counterexample to the latter conjecture. His random field is not a Gibbs state in the usual sense: it involves constraints rather than interactions. One could construct an interaction whose Gibbs states have similar behaviour, but it would have to be unbounded, growing rapidly enough at infinity so that with high probability all the constraints of von Weizsäcker's process are satisfied. In this note we will construct examples with a bounded, in fact periodic or translation-invariant, interaction whose extremal Gibbs states fail to have the global Markov property.

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